1985 AP Calculus AB: Section I

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

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2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

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3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

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4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

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5. $\lim_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

6. If $f(x) = x$, then $f'(5) =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

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7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\, dt$ (D) $\int_1^4 \ln x\, dx$ (E) $\int_1^4 \frac{1}{t}\, dt$

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8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

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9. If $\int_{-1}^{1} e^{-x^2}\, dx = k$, then $\int_{-1}^{0} e^{-x^2}\, dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

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10. If $y = 10^{\left(x^2-1\right)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

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11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

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12. If $f\left(g(x)\right) = \ln\left(x^2+4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\dfrac{dy}{dx} =$

(A) $-\dfrac{2x+y}{x+3y^2}$ (B) $-\dfrac{x+3y^2}{2x+y}$ (C) $\dfrac{-2x}{1+3y^2}$ (D) $\dfrac{-2x}{x+3y^2}$ (E) $-\dfrac{2x+y}{x+3y^2-1}$

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14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

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15. The domain of the function defined by $f(x) = \ln\left(x^2 - 4\right)$ is the set of all real numbers x such that

(A) $|x| < 2$ (B) $|x| \le 2$ (C) $|x| > 2$ (D) $|x| \ge 2$ (E) x is a real number

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16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

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17. $\int_0^1 xe^{-x}\,dx =$

(A) $1-2e$ (B) -1 (C) $1-2e^{-1}$ (D) 1 (E) $2e-1$

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18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

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19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x) = x+1$ (B) $f(x) = 2x$ (C) $f(x) = \dfrac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2 + 1}$ (E) $\frac{1}{1+\cos^2 x}$

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21. If the domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of f?

(A) $\{x : -\infty < x < -1\}$ (B) $\{x : -\infty < x < 0\}$ (C) $\{x : -\infty < x < 1\}$

(D) $\{x : -1 < x < \infty\}$ (E) $\{x : 0 < x < \infty\}$

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22. $\int_1^2 \frac{x^2-1}{x+1}\, dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

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23. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

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24. If $\int_{-2}^{2} \left(x^7 + k\right) dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

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25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h \to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h \to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h \to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h \to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h \to 0} \frac{e^{e+h} - e^e}{h}$

26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

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27. $\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

---

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) −45 (B) −30 (C) −15 (D) −10 (E) −5

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29. Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

---

30. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)| + C$ (B) $-\frac{1}{2}\ln|\cos(2x)| + C$ (C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

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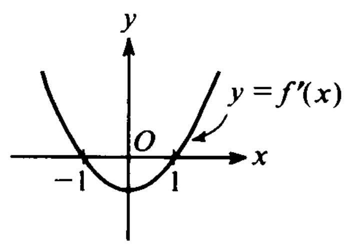
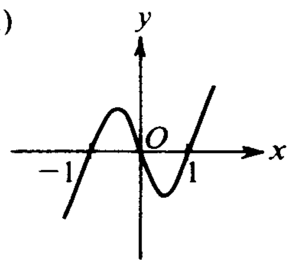
32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

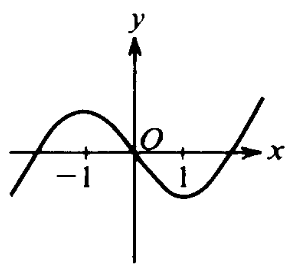


33. The graph of the <u>derivative</u> of f is shown in the figure above. Which of the following could be the graph of f?

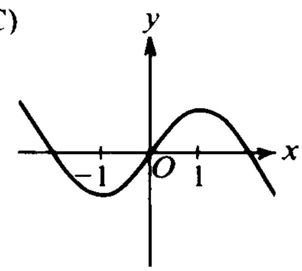
(A)



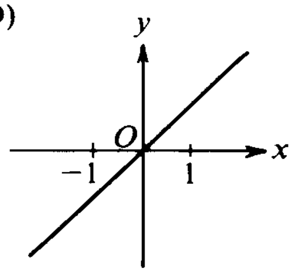
(B)



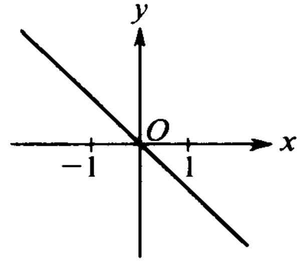
(C)



(D)

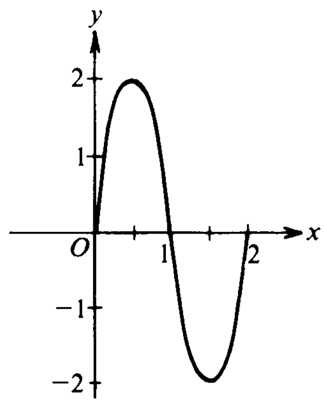


(E)



34. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$



35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

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36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

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37. $\lim_{x \to 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

I. $f'(x) \le g'(x)$ for all real x

II. $f''(x) \le g''(x)$ for all real x

III. $\int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

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39. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

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40. Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

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41. If $\lim_{x\to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above

42. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

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43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

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44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0, 2]$ is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

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45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

1985 Answer Key

1985 AB

1. D
2. E
3. A
4. C
5. D
6. C
7. E
8. B
9. D
10. D
11. B
12. C
13. A
14. D
15. C
16. B
17. C
18. C
19. B
20. A
21. B
22. A
23. B
24. D
25. E
26. E
27. D
28. C
29. D
30. B
31. C
32. D
33. B
34. A
35. D
36. B
37. D
38. C
39. E
40. D
41. E
42. C
43. B
44. A
45. A

1985 BC

1. D
2. A
3. B
4. D
5. D
6. E
7. A
8. C
9. B
10. A
11. A
12. A
13. B
14. C
15. C
16. C
17. B
18. C
19. D
20. C
21. B
22. A
23. C
24. D
25. C
26. E
27. E
28. E
29. D
30. B
31. D
32. E
33. C
34. A
35. B
36. E
37. A
38. C
39. A
40. A
41. C
42. E
43. E
44. A
45. D

1. D $\int_1^2 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$.

2. E $f'(x) = 4(2x+1)^3 \cdot 2$, $f''(1) = 4\cdot 3(2x+1)^2 \cdot 2^2$, $f'''(1) = 4\cdot 3\cdot 2(2x+1)^1 \cdot 2^3$,
$f^{(4)}(1) = 4!\cdot 2^4 = 384$

3. A $y = 3(4+x^2)^{-1}$ so $y' = -3(4+x^2)^{-2}(2x) = \dfrac{-6x}{(4+x^2)^2}$

Or using the quotient rule directly gives $y' = \dfrac{\left(4+x^2\right)(0)-3(2x)}{\left(4+x^2\right)^2} = \dfrac{-6x}{\left(4+x^2\right)^2}$

4. C $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)\,(2\,dx) = \frac{1}{2}\sin(2x)+C$

5. D $\lim\limits_{n\to\infty} \dfrac{4n^2}{n^2+10000n} = \lim\limits_{n\to\infty} \dfrac{4}{1+\dfrac{10000}{n}} = 4$

6. C $f'(x) = 1 \Rightarrow f'(5) = 1$

7. E $\int_1^4 \frac{1}{t}\,dt = \ln t\Big|_1^4 = \ln 4 - \ln 1 = \ln 4$

8. B $y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$, $y' = \frac{1}{x}$, $y'(4) = \frac{1}{4}$

9. D Since e^{-x^2} is even, $\int_{-1}^0 e^{-x^2}\,dx = \frac{1}{2}\int_{-1}^1 e^{-x^2}\,dx = \frac{1}{2}k$

10. D $y' = 10^{(x^2-1)}\cdot \ln(10)\cdot \frac{d}{dx}\left((x^2-1)\right) = 2x\cdot 10^{(x^2-1)}\cdot \ln(10)$

11. B $v(t) = 2t+4 \Rightarrow a(t) = 2 \therefore a(4) = 2$

12. C $f\left(g(x)\right) = \ln\left(g(x)^2\right) = \ln\left(x^2+4\right) \Rightarrow g(x) = \sqrt{x^2+4}$

13. A $2x + x\cdot y' + y + 3y^2\cdot y' = 0 \Rightarrow y' = -\dfrac{2x+y}{x+3y^2}$

14. D Since $v(t)\ge 0$, distance = $\int_0^4 \left|v(t)\right| dt = \int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}\right) dt = \left(2t^{\frac{3}{2}} + 2t^{\frac{5}{2}}\right)\Big|_0^4 = 80$

15. C $x^2 - 4 > 0 \Rightarrow |x| > 2$

16. B $f'(x) = 3x^2 - 6x = 3x(x-2)$ changes sign from positive to negative only at $x = 0$.

17. C Use the technique of antiderivatives by parts:

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$-xe^{-x} + \int e^{-x}\,dx = \left(-xe^{-x} - e^{-x}\right)\Big|_0^1 = 1 - 2e^{-1}$

18. C $y = \cos^2 x - \sin^2 x = \cos 2x$, $y' = -2\sin 2x$

19. B Quick solution: lines through the origin have this property.

Or, $f(x_1) + f(x_2) = 2x_1 + 2x_2 = 2(x_1 + x_2) = f(x_1 + x_2)$

20. A $\dfrac{dy}{dx} = \dfrac{1}{1+\cos^2 x}\cdot\dfrac{d}{dx}(\cos x) = \dfrac{-\sin x}{1+\cos^2 x}$

21. B $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow f(x) < 0$ for all x in the domain. $\lim\limits_{|x|\to\infty} f(x) = 0$. $\lim\limits_{|x|\to 1} f(x) = -\infty$. The only option that is consistent with these statements is (B).

22. A $\int_1^2 \dfrac{x^2-1}{x+1}\,dx = \int_1^2 \dfrac{(x+1)(x-1)}{x+1}\,dx = \int_1^2 (x-1)\,dx = \dfrac{1}{2}(x-1)^2\Big|_1^2 = \dfrac{1}{2}$

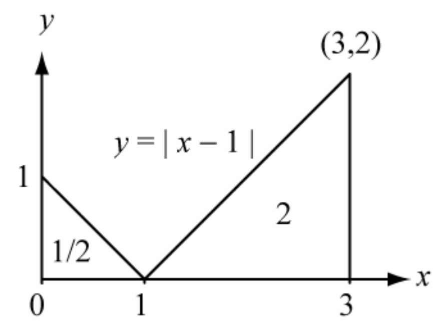
23. B $\dfrac{d}{dx}\left(x^{-3} - x^{-1} + x^2\right)\Big|_{x=-1} = \left(-3x^{-4} + x^{-2} + 2x\right)\Big|_{x=-1} = -3 + 1 - 2 = -4$

24. D $16 = \int_{-2}^{2}(x^7 + k)\,dx = \int_{-2}^{2} x^7\,dx + \int_{-2}^{2} k\,dx = 0 + \left(2 - (-2)\right)k = 4k \Rightarrow k = 4$

25. E $f'(e) = \lim_{h \to 0} \frac{f(e+h) - f(e)}{h} = \lim_{h \to 0} \frac{e^{e+h} - e^e}{h}$

26. E I: Replace y with $(-y)$: $(-y)^2 = x^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
II: Replace x with $(-x)$: $y^2 = (-x)^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
III: Since there is symmetry with respect to both axes there is origin symmetry.

27. D The graph is a V with vertex at $x = 1$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 0 to 3. These triangles have areas of 1/2 and 2 respectively.



28. C Let $x(t) = -5t^2$ be the position at time t. Average velocity $= \frac{x(3) - x(0)}{3 - 0} = \frac{-45 - 0}{3} = -15$

29. D The tangent function is not defined at $x = \pi/2$ so it cannot be continuous for all real numbers. Option E is the only one that includes item III. In fact, the functions in I and II are a power and an exponential function that are known to be continuous for all real numbers x.

30. B $\int \tan(2x)\,dx = -\frac{1}{2}\int \frac{-2\sin(2x)}{\cos(2x)}\,dx = -\frac{1}{2}\ln\left|\cos(2x)\right| + C$

31. C $V = \frac{1}{3}\pi r^2 h$, $\frac{dV}{dt} = \frac{1}{3}\pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right) = \frac{1}{3}\pi\left(2(6)(9)\left(\frac{1}{2}\right) + 6^2\left(\frac{1}{2}\right)\right) = 24\pi$

32. D $\int_0^{\pi/3} \sin(3x)\,dx = -\frac{1}{3}\cos(3x)\Big|_0^{\pi/3} = -\frac{1}{3}\left(\cos\pi - \cos 0\right) = \frac{2}{3}$

33. B f' changes sign from positive to negative at $x = -1$ and therefore f changes from increasing to decreasing at $x = -1$.

Or f' changes sign from positive to negative at $x = -1$ and from negative to positive at $x = 1$. Therefore f has a local maximum at $x = -1$ and a local minimum at $x = 1$.

34. A $\int_0^1 \left((x+8) - (x^3+8)\right)dx = \int_0^1 (x - x^3)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{4}$

35. D The amplitude is 2 and the period is 2.

$y = A\sin Bx$ where $|A| = \text{amplitude} = 2$ and $B = \dfrac{2\pi}{\text{period}} = \dfrac{2\pi}{2} = \pi$

36. B II is true since $|-7| = 7$ will be the maximum value of $|f(x)|$. To see why I and III do not have to be true, consider the following: $f(x) = \begin{cases} 5 & \text{if} \quad x \le -5 \\ -x & \text{if} \quad -5 < x < 7 \\ -7 & \text{if} \quad x \ge 7 \end{cases}$

For $f(|x|)$, the maximum is 0 and the minimum is –7.

37. D $\lim\limits_{x\to 0} x\csc x = \lim\limits_{x\to 0} \dfrac{x}{\sin x} = 1$

38. C To see why I and II do not have to be true consider $f(x) = \sin x$ and $g(x) = 1 + e^x$. Then $f(x) \le g(x)$ but neither $f'(x) \le g'(x)$ nor $f''(x) < g''(x)$ is true for all real values of x.

III is true, since

$f(x) \le g(x) \Rightarrow g(x) - f(x) \ge 0 \;\Rightarrow\; \int_0^1 \big(g(x) - f(x)\big)\,dx \ge 0 \Rightarrow \int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

39. E $f'(x) = \dfrac{1}{x}\cdot\dfrac{1}{x} - \dfrac{1}{x^2}\ln x = \dfrac{1}{x^2}(1 - \ln x) < 0$ for $x > e$. Hence f is decreasing. for $x > e$.

40. D $\int_0^2 f(x)\,dx \le \int_0^2 4\,dx = 8$

41. E Consider the function whose graph is the horizontal line $y = 2$ with a hole at $x = a$. For this function $\lim\limits_{x\to a} f(x) = 2$ and none of the given statements are true.

42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x) = \sqrt{1 + x^2}$

43. B $y' = 3x^2 + 6x$, $y'' = 6x + 6 = 0$ for $x = -1$. $y'(-1) = -3$. Only option B has a slope of –3.

44. A $\dfrac{1}{2}\int_0^2 x^2\left(x^3+1\right)^{1/2} dx = \dfrac{1}{2}\cdot\dfrac{1}{3}\int_0^2 \left(x^3+1\right)^{1/2}\left(3x^2 dx\right) = \dfrac{1}{6}\left(x^3+1\right)^{3/2}\cdot\dfrac{2}{3}\Bigg|_0^2 = \dfrac{26}{9}$

45. A Washers: $\sum \pi\left(R^2 - r^2\right)\Delta y$ where $R = 2$, $r = x$

$$\text{Volume} = \pi\int_0^4 \left(2^2 - x^2\right)dy = \pi\int_0^4 (4 - y)\,dy = \pi\left(4y - \frac{1}{2}y^2\right)\Bigg|_0^4 = 8\pi$$